**Semester One Examination, 2018**

**Question/Answer booklet**

## MATHEMATICS SPECIALIST UNIT 3,4

**Section One:**
**Calculator-free**

If required by your examination administrator, please place your student identification label in this box

Student number:    In figures

In words    _____

Your name    _____

## Time allowed for this section
Reading time before commencing work:     five minutes
Working time:     fifty minutes

## Materials required/recommended for this section
***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet

***To be provided by the candidate***
Standard items:    pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:    nil

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 53 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1.  The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.  Write your answers in this Question/Answer booklet.

3.  You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.  Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5.  Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.  It is recommended that you do not use pencil, except in diagrams.

7.  The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**

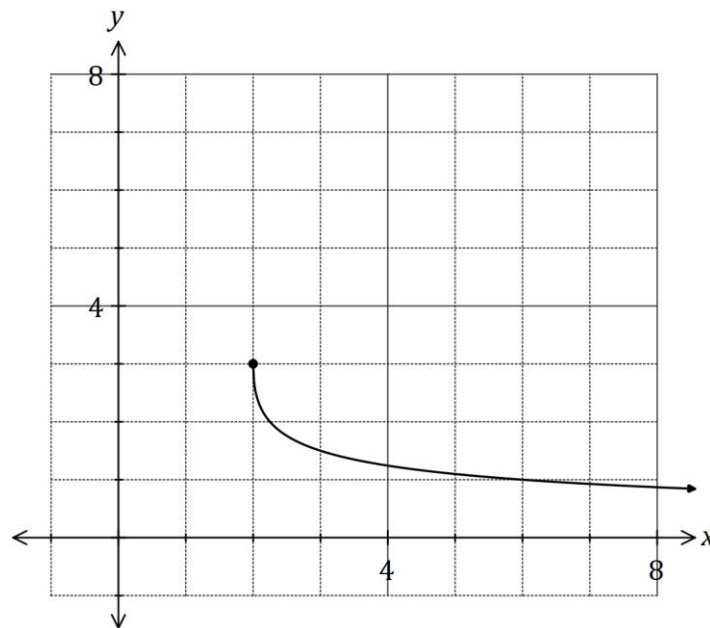**Section One: Calculator-free**  **35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1**  **(6 marks)**

The graph of $y = f(x)$ is shown below, where $f$ is defined by $f(x) = \dfrac{3}{1 + \sqrt{x-2}}$ .



(a)  Sketch the graph of $y = f^{-1}(x)$ on the same axes.  (2 marks)

(b)  Determine the defining rule for $y = f^{-1}(x)$ and state its domain.  (4 marks)

**Question 2**                                                      **(7 marks)**

Consider $f(z) = 5z^3 + 2z^2 + 10z + 4$, where $z$ is a complex number.

(a)     Determine, with reasons, which of the following are factors of $f(z)$.

      (i)      $z - 2$.                                                     (2 marks)
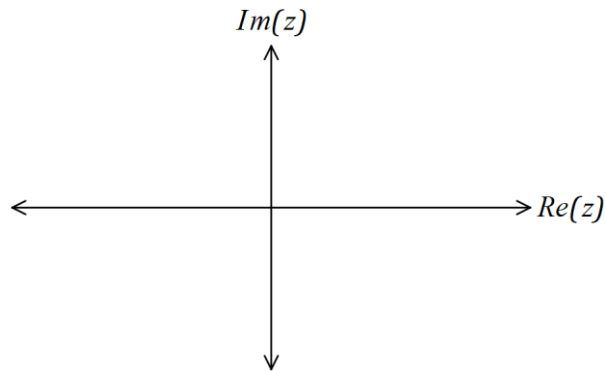
      (ii)     $z - \sqrt{2}i$.                                               (2 marks)

(b)     Solve the equation $f(z) = 0$.                                (3 marks)

**Question 3**                                                                    **(7 marks)**

(a)     Locate the roots of the complex equation $z^4 - 1 = 0$ in the Argand plane below.  (3 marks)



(b)     State the sum of all the roots of the complex equation $z^4 - 1 = 0$.                (1 mark)

(c)     Let $u$ be any 4th root of unity, where $\operatorname{Im} u \neq 0$.

Show that $u(1 + 5u)(1 + 5u^2) = 4(5 - u)$.                                        (3 marks)

**See next page**

**Question 4**        **(6 marks)**

(a)    Solve this system of equations.        (3 marks)

$$x + y + 2z = 1$$
$$4x + y - z = 7$$
$$3x - y + z = 14$$

(b)    Determine the value of constant $a$ so that the following system of equations does not have a unique solution and give a brief geometric interpretation of the system of equations with this value.        (3 marks)

$$x + y + 2z = 1$$
$$4x + y - z = 7$$
$$ax - y + z = 14$$

**Question 5** **(10 marks)**

The points $A$, $B$ and $C$ have position vectors $(1, 0, -2)$, $(b, -2, 1)$ and $(2, -1, 0)$ respectively.

(a)     Determine the vector equation for the line through $A$ and $C$.     (2 marks)

(b)     Determine, in terms of $b$, the Cartesian equation of the plane containing $A$, $B$ and $C$.
                                                                        (5 marks)

(c)     The line with equation $\mathbf{r} = (3, -5, 6) + \mu(2, q, -12)$ is perpendicular to the plane containing $A$, $B$ and $C$. Determine the values of the constants $b$ and $q$.     (3 marks)

**Question 6**                                                                               **(5 marks)**
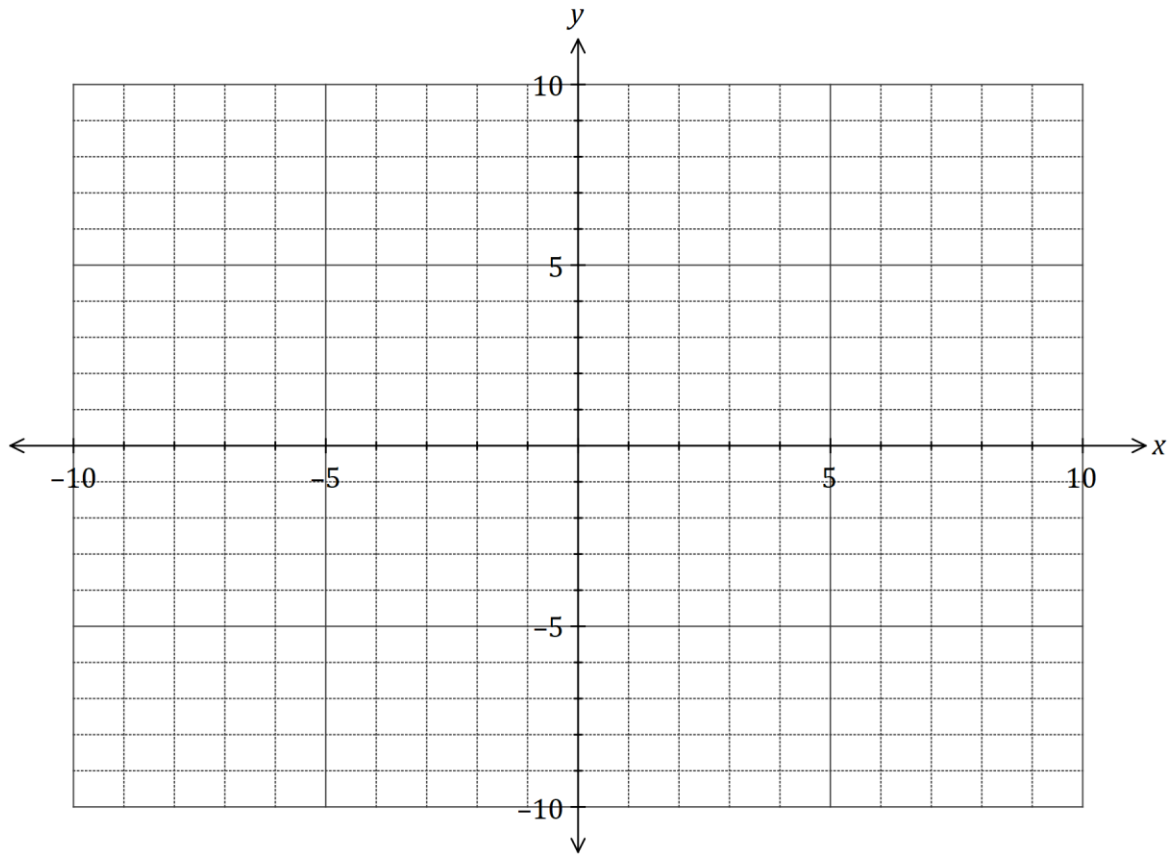
The complex numbers $u$ and $v$ satisfy the equations $u - v = 2i$ and $uv = 10$.

Solve the equations for $u$ and $v$, giving your solution(s) in the form $x + yi$, where $x$ and $y$ are real.

**Question 7** (6 marks)

The graph of $y = \dfrac{3x^2 - 12}{(x + 1)(x - 3)}$ has no stationary points. Sketch the graph.

**Question 8** (6 marks)

A function is defined by $f(x) = \dfrac{x + 4}{(3x + 2)(5x - 4)}$ .

(a) State the natural domain of $f(x)$. (1 mark)

(b) State the equations of all asymptotes of the graph of $y = x \cdot f(x)$. (2 marks)

(c) The graph of $y = \dfrac{1}{f(x)}$ has an asymptote with equation $y = ax + b$. Determine the values of the constants $a$ and $b$. (3 marks)

**End of questions**

Supplementary page

Question number: _____